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Prof. Saul Teukolsky
Hans A. Bethe Professor of Physics and Astrophysics
608 Space Sciences Building
Cornell University
Itaca, NY 14853
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Dear Professor Teukolsky,

I have read with interest the 30-page letter by Dr. E. J. Weniger that is posted at the website of the third edition of your book *Numerical Recipes*. The letter expresses satisfaction from the fact that you have included a chapter on convergence acceleration (or extrapolation) methods in your book, and goes on to expressing some personal opinions about the subject in general and the relevant literature in particular. When expressing his opinion about the literature, Weniger chooses to concentrate only on my book

Practical Extrapolation Methods: Theory and Applications

and makes remarks about it, some of which I find not correct. (The same remarks and more were made in his earlier reviews of my book as well; see references [31,32] in Weniger's letter.)
Weniger claims that

1. my book is a book written by an expert ... for the few other ones who can also claim to be experts in this field,
2. my book is hard to read even for specialists,
3. my choice of topics, and in some cases also the deliberate omission of certain topics, makes this book highly subjective,
4. non-specialist readers may get a distorted view about the state of the art and the contributions of other researchers.

In response, I would like to make the following clarifications concerning these claims, in the order listed above:

1. My book was written keeping in mind the needs of both the theoretician and the practitioner. While I was very rigorous in my theoretical treatment of the various methods, I was also very careful to provide the practitioner with enough motivation, many examples, well documented algorithms, and detailed explanations on how to use each extrapolation method for best results in floating-point arithmetic. Furthermore,

my book contains several chapters dealing exclusively with the treatment of oscillatory integrals, power series, Fourier series and generalized Fourier series, all arising from common scientific and engineering problems, and of immediate interest to the practitioner. Finally, the 40-page-long Chapter 25 in my book provides a whole sequence of additional applications that will be of interest to the practitioner, as well as the theoretician.

2. In writing my book, my main purpose was also to provide a rigorous mathematical presentation of the state of the art in extrapolation. I specifically did not want to compromise mathematical rigor in favor of "easy reading." I am sure this will be appreciated by any serious researcher and student of this important field.
   Having made this comment, let me add that to grasp the theory one is required to have reached a certain degree of mathematical maturity. Thus, the expert should not have any difficulty when studying my book. Similarly, an advanced undergraduate of mathematics, physics, or engineering who has taken enough calculus courses and has studied some asymptotics (a must!) should not have much difficulty in learning the subject of extrapolation by studying my book, provided, of course, he is willing to take the subject seriously. As for the practitioner who is more interested in applying the methods, he can benefit a lot by following the arguments, algorithms, and examples, without having to go into the theory in any depth. I was informed by some non-expert colleagues that they had learned quite a bit from my book.

3. My choice of topics (methods, algorithms, analyses, and applications) is more comprehensive than in previous works, and this certainly makes my book more objective than others. As far as methods of extrapolation go, I chose to include all those methods that were proved to be efficient in the various review papers, e.g., two well known papers of Smith and Ford (references [76,77] in Weniger's letter) and a paper of Weniger (reference [81] in Weniger's letter). I left out the methods that were not as efficient as those that passed through the filters of these papers. I also concentrated on methods that are *practical*, in the sense that they have large scopes, such as $\mathrm{GREP}^{(m)}$, the D, $\overline{D}$, W, and mW transformations for infinite-range integrals and the d transformation for infinite series. I devoted a 25-page chapter to Padé approximants and the developments that took place in them until recently and that are relevant to convergence acceleration) and a 15-page chapter to some of their generalizations that serve as extrapolation methods. I gave the most complete treatments of well known methods such as the classical Richardson extrapolation method, Euler's transformation, Lubkin's transformation, Aitken's Δ^2 process, Shanks' transfor-

mation, Levin's transformations, the ρ algorithm, the θ algorithm, and the high-order G transformation, to name some. Thus, the number of methods and applications discussed in my book is much larger than that in any other book that preceded mine. Since my book already contains almost everything considered in previous works, to claim that it is subjective is simply wrong.

4. By studying my book, especially the examples in it, non-specialists will get a very good idea about the state of the art in extrapolation methods. They will not be wondering about what to do with their problem when the method of extrapolation they use fails. They will learn how to analyze a given sequence *qualitatively* (that is, without applying any deep mathematics) to decide which method should be used, how it should be used, and how it should be tuned for best results in floating-point arithmetic. In this respect, I would like to refer the interested reader to Appendix H in my book. In this appendix, (i) the types of sequences and series that arise in most applications are classified in a way that can easily be understood by non-experts, and (ii) the extrapolation methods that can be applied with success to each and every one of these types are pointed out. Appendix I of my book illustrates this abundantly with a FORTRAN 77 program implementing the d transformation and applying it to 15 examples of different types! All this is definitely missing from previous works.

Finally, I would like to remark on the opinion that features in Weniger's letter, according to which, the topic of convergence acceleration is an "experimental science," in the sense that one has to try more than one method to have confidence in the numerical results. This used to be true until thirty years ago, but is not true any longer. Today, we know, by a cursory look at a series or an integral, which method we should use and how we should use it, as I already explained above. Thus, the topic of extrapolation methods is a "true science." It can and should be explored and studied systematically, precisely as is done in my book.

To get a balanced picture about my book, I invite the readers to take a look at the reviews that appeared in *Mathematical Reviews* and *Zentralblatt* and at the review article by Prof. David Levin, himself a most famous figure in extrapolation theory. This review appeared in *Mathematics of Computation*, Volume 74, No. 250, 2005.

Sincerely yours,

Avram Sidi